Application note

LU and LDL^T factorization in SSN

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Contents

Document revision history

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1. Executive summary

This document objective is to explain the SSN factorization options LU and LDL^T.

The LDL^T factorisation can bring significant speed increases in SSN, especially in models with larger number of SSN nodes. The speed gain is 2 in theory.

LDL^T factorisation suffers however from 2 limitations:

1- As LDL^T assumes a symmetric admittance matrix, it cannot be used with models that produces asymmetric admittance matrices. Several SSN machines can produce such asymmetric matrices. These machines includes a ‘delayed speed term’ option that can circumvent this issue.

2- Again because it assumes a symmetry in the admittance matrix, the LDL^T is more numerically sensitive than LU factorization. This sensitivity occurs in SSN models or groups that are badly conditioned numerically. In the process of obtaining the SSN matrices for such model or group, it can happen that some numerical inverse produces slightly asymmetric results, which will cause the global SSN admittance matrix to be asymmetric too.

Recent versions of ARTEMiS can printout the worst condition number (rcond) of the SSN groups to help to detect such conditions.

2. Software Requirement

To perform the parameter modification using the method described in this documentation, the following software is required to properly install on a computer that is used to conduct this operation.

Table 1 List of Software required

Software Names	Versions
MATLAB	R2015a

3. LU vs. LDL^T factorisation

LU and LDL^T factorisations are used to solve systems of linear equations.

$$Ax = b \rightarrow LUx = b \;\; or \;\; LDL^T x = b$$

Where L is a lower triangular matrix and U is an upper triangular matrix. In the LDL^T method, D is a diagonal matrix and L^T is the transposed of L.

With lower, upper and diagonal matrices, the solution to this matrix problem can be found line by line with forward and backward substitutions.

The LDL^T factorisation is a variant of Cholesky factorisation where the D matrix do not appear. RTDS reportedly uses the Cholesky method according to their own papers.

3.1 Use with power system admittance matrices

A condition to use the LDL^T factorisation is that the A matrix be symmetric.

Additionally, real-time implementation of these factorisation method will avoid pivoting. Pivot is NOT required when the matrix to be factorised is diagonally dominant, which is usually the case in power systems.

Power system admittance matrices produced by the classic EMTP method (used in Hypersim and SSN but not in EMTP-RV however) are most often diagonally dominant and symmetric. The dominance of the diagonal avoids pivoting during LU and LDL^T factorisations.

The old EMTP also produces symmetric nodal admittance matrices, a condition to use LDL^T factorisation. In SSN, this condition is not always met however: SSN Machines models produce asymmetric admittance matrices when the standard high-precision model is used. The SSN machine comes with the 'Delayed Speed term' option that produces symmetric matrices at the cost of an added delay in the speed term of the machine equations.

3.2 Speed of calculation

3.2.1 Theory

According to https://en.wikipedia.org/wiki/Cholesky_decomposition , the algorithms described here involve about $n^3/3$ multiplications and additions, where n is the size of the matrix A. Hence, they have half the cost of the LU decomposition, which uses $2n^3/3$ multiplications and additions [2].

These speeds can with implementations but grosso modo the LDL^T factorisation is twice as fast as LU.

3.2.2 SSN measurements

In SSN, the LDL^T factorisation produces speed gains.

The Nine-level motor drive with multi-winding zig-zag feeding transformer (SSN) demo has 39 SSN nodes mainly used to compute the 3*4=12 diode rectifiers (72 switches). A similar model with 17-level inverter with multi-winding zig-zag feeding transformer was designed for a client (confidential) in which there is 76 SSN nodes and a total of 3*8=24 diode rectifiers (144 switches)

In these models, the factorization part of the SSN algorithm is dominant by factors of 1.5 (39 nodes) and 3 (76 nodes) and we compare below the calculation time of both LU and LDL^T factorization methods.

TABLE 1: Total calculation time on Opal-RT OP4510 in micro-seconds. 39 node model, 72 switches, 2 core available for the rectifiers

	LU	LDL^T
1 core	42.8	31.2
2 cores	38.0	26.3

TABLE 2: Total calculation time on Opal-RT OP4510 in micro-seconds. 76 node model, 144 switches, 3 cores available for the rectifiers

	LU	LDL^T
1 core	205.9	133.4
2 cores	195.5	125.6
3 cores	183.3	114.5

These timings are the total model timing, not only the factorisation part. They show a notable speed increase by the LDL^T method.

3.3 LDL^T and SSN machine models

Most SSN machine models (except the rotor-cage induction machine) produce by default asymmetrical admittance matrix because of the inclusion of the speed term in the A state matrix, in the ABCD formulation.

It is possible to include the speed term in the B matrix as well, with a delay. All SSN machines provides this ‘delayed speed term’ option. The SSN machines also check for the LU factorisation to be selected when the ‘delayed speed term’ is unselected.

3.3.1 Accuracy impacts

From the author experience, the ‘delayed speed term’ has minimal accuracy impact on grid power systems running at 50/60Hz. But the accuracy can become bad with machine models running at 400/800Hz, such as ones found on aircraft for example. Users should verify the accuracy by themselves on such high-speed cases.

3.4 Numerical sensitivity in SSN

In SSN, it was found that the LDL^T factorisation was more susceptible to numerical problems than LU in some cases.

The typical cause is a badly conditioned SSN group with almost non-invertible admittance matrix. This case actually happens in the case of a SSN groups with all I-type NIB and no ground reference. Such a group is almost isolated from ground and the reverse of the admittance matrix may become asymmetric for *numerical reasons.* The interesting aspect now is that the LU solution is more tolerant to this type of numerical issue.

Detecting such cases is difficult. In SSN, this situation can be detected using the m-file S-function (non real-time), by setting >> USE_MFILE_SSN_SFUNCTION=1 in the MATLAB workspace. This is done in the next example.

3.4.1 Bipolar HVDC demo example

The ‘***Bipolar 12-pulse HVDC link with switched filter banks’*** demo (ssn_Bipolarhvdc12pSwFilterBanks.mdl) is used here to illustrate this possible numerical sensitivity. In this model there is a SSN group made out of the upper valves of the rectifier stations. If we set this group to have all I-type interface with the LDL^T factorisation set, we get some warning in the Diagnostics pane, signaling that a group has a very low rcond value:

```
... SSN group info
Group 1 : 5 states, 9 inputs, 15 outputs, 0 switches.
Group 2 : 5 states, 9 inputs, 15 outputs, 0 switches.
Group 3 : 21 states, 6 inputs, 3 outputs, 0 switches.
Group 4 : 6 states, 6 inputs, 3 outputs, 0 switches.
Group 5 : 1 states, 3 inputs, 6 outputs, 1 switches.
Group 6 : 1 states, 3 inputs, 6 outputs, 1 switches.
Group 7 : 6 states, 22 inputs, 11 outputs, 6 switches. (WARNING: low Y rcond= 1.6664e-13 )
Group 8 : 21 states, 9 inputs, 6 outputs, 3 switches. (Y rcond= 5.8525e-05 )
Group 9 : 21 states, 9 inputs, 6 outputs, 3 switches. (Y rcond= 5.8525e-05 )
Group 10 : 21 states, 9 inputs, 6 outputs, 3 switches. (Y rcond= 5.8525e-05 )
Group 11 : 21 states, 9 inputs, 6 outputs, 3 switches. (Y rcond= 5.8525e-05 )
Group 12 : 21 states, 9 inputs, 6 outputs, 3 switches. (Y rcond= 5.8525e-05 )
Group 13 : 21 states, 9 inputs, 6 outputs, 3 switches. (Y rcond= 5.8525e-05 )
Group 14 : 6 states, 16 inputs, 10 outputs, 6 switches.
Group 15 : 6 states, 17 inputs, 11 outputs, 6 switches.
Group 16 : 6 states, 16 inputs, 10 outputs, 6 switches.
SSN nodal matrix is of rank 19 (45.4294 % of zeros) (0 prefactorized col/rows)
...
```

Rcond is Reverse Conditioning number and indicates a near singular matrix when value is close to 0. Simulation of this model in these condition becomes inaccurate quickly with LDL^T factorization. The LU factorization is fine in this case but user should nevertheless correct any low rcond warnings.

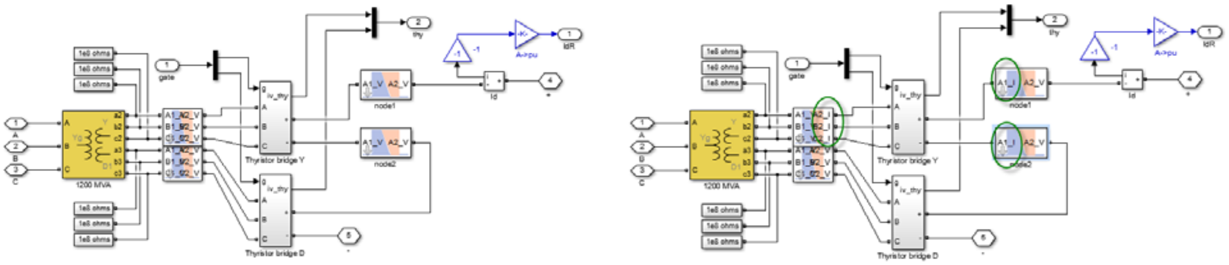

Figure 1: Bipolar HVDC demo with I-type NIB inteface on upper valve group (Left: standard, right: I-type version)

4. References

[1] Christian Dufour , “Highly stable rotating machine models in the state-space-nodal solver”, unpublished, available in the Scientific Paper section of ARTEMiS.

[2] Trefethen, Lloyd N.; Bau, David (1997). Numerical linear algebra. Philadelphia: Society for Industrial and Applied Mathematics. ISBN 978-0-89871-361-9.