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An Advanced Real-Time Electro-Magnetic Simulator for Power Systems with a Simultaneous State-Space Nodal Solver

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***Abstract*— This paper presents a simulation method that combines state-space analysis with a nodal method for the simulation of electrical systems. This paper extends the concept of a discrete companion branch equivalent of the nodal approach to state-space described systems, and enables natural coupling between them. The flexible clustering of state-space described electrical subsystems into a nodal method has the following advantages: first, the nodal admittance matrix can be constrained in size while still permitting the solution of a switched network by nodal admittance matrix on-line triangularisation. Also, each group can have a precalculation of all internal modes (caused by switches, for example) within itself, an important feature for real-time applications. Secondly, the state-space formulation enables the use of higher-level discretization methods with L-stability properties. Finally, the approach enables the coupling of complex nodal-based models like FD-line into a state-space based solver. The method is implemented in a commercial real-time simulation software tool, the Advanced Real-Time Electro-Magnetic Simulator (ARTEMiS).**

***Index Terms*— EMTP, Real-Time Simulation, solvers, state-space, state-space nodal, ARTEMiS.**

I. INTRODUCTION

The EMTP program is the most widely used software for simulating power networks [1][2]. In EMTP, basic groups consisting of R, L, C, machines and lines are discretized with the trapezoidal method, an A-stable implicit method. A-stability provides for stability of simulation of sometimes stiff networks. This implicitness means that, for each branch, the solution cannot be found uniquely from the previous states and current input values. This is because nodal voltage points of all branches have a common solution that must be solved simultaneously for all groups/branches. To solve this problem, companion branch equations with a Voltage/Current or Current/Voltage relationship are defined. This admittance/susceptance relationship, called 'discrete resistance', together with explicit terms known from previous time-steps, can then be combined into a nodal matrix to simultaneously solve the unknown nodal voltages and branch current of all elements.

The original EMTP method makes use only of simple RLC elements that are individually inserted and solved in a large admittance matrix. Interesting variations of the approach exist in which some groups are formed prior to the inclusion into the admittance matrix, like in MATE[4] and GENE [6]. The use of groups provides a way to diminish the number of nodal points in the composition of the global admittance matrix. This, in turn, can lead to improvements in calculation time, which is important especially in real-time applications. [13] proposes a similar method based on the EMTP 'compensation method'.

The method proposed in this paper is a nodal method based on pre-concatenation of basic RLC elements into the admittance matrix of the nodal method. However, the way that group equations are derived is different. In the proposed method, the complete network simulation equations are explicitly and uniformly derived from the state-space equation of each of the groups. The method is explained in more detail in [12].

The main objective of this paper is to explain the mathematical background of the approach and the various challenges faced with the method for implementation in a real-time simulator. In this regard, issues related to switches, numerical instability of the trapezoidal rule, compensation of switching events and the possibility to parallelize the algorithm will be examined.

The proposed method will be validated on several models: an academic switched RLC system, a complete 12-pulse HVDC system with switched AC filter banks, a Static Voltage Compensator (SVC) and a small distribution system for breaker tests. The method is also used to incorporate a Frequency Dependent Parameter line model, coded with the nodal approach, into the state-space method of SimPowerSystems. Comparison will be made against SimPowerSystems (SPS) and EMTP-RV, which use the trapezoidal rule of integration and also against the order 5 discretisation method of ARTEMiS [3] [8].

II. STATE-SPACE NODAL METHOD

The state-space nodal method described hereafter is a generalisation of the Dommel method that uses arbitrary sized state-space described clusters of electrical elements and combines them with the nodal admittance matrix using an EMTP-like nodal method.

In the classic EMTP method, a fixed set of basic elements (such as R, series RL, series RLC, transformer) are discretized individually. Their discretized equivalents are solved simultaneously with the use of a nodal admittance matrix. Because the basic elements are small and numerous, the resulting nodal matrix can be huge and therefore pose a challenge during real-time simulation. In the MATE method for example, some pre-grouping of the simulated network is made prior to incorporation into the nodal matrix. This has the important effect of producing a smaller nodal matrix, therefore facilitating real-time simulation. In these methods, the reduction of group equations is made by RLC branch reduction of the discretized equivalents. The proposed method of this paper also makes use of pre-grouping of the network to be simulated to obtain a reduced size nodal matrix. The main difference between this and the two previous methods is that the group equations are derived directly from their state-space equations.

In the proposed method, the discrete equations of all groups is directly derived from their ABCD state-space representation. From the continuous Laplace state-space equations, the discrete equations are first found through the use of the trapezoidal or higher order rule of integration. These groups' discrete equations include implicitly unknown entries because these entries, the nodal voltage, are common to all groups and must be solved simultaneously.

However, from these discrete equations an impedance/admittance ratio can be found at the current time-step between the voltage entries and the current input (or vice-versa depending on the choice of entry type for the state-space system). These ratios, commonly referred to as 'discrete resistance' or 'discrete admittance' in the literature, are then used to build a global admittance network from which the a priori unknown nodal voltage values can be found simultaneously for all groups. With this nodal solution, each group can complete its iteration for the current time-step.

A. State-space modeled groups

Suppose there is a group of resistance, inductance, capacitance and transformer sources, and other electrical elements connected to a terminal of unknown voltage and current (a 'nodal connection point'). The state-space equation exists:

$$\begin{aligned} x' &= A_k x + B_k u \\ y &= C_k x + D_k u \end{aligned} \quad (1)$$

where

x: states of the system
u: inputs of the system
y: output of the system

A_k , B_k , C_k , D_k : state space matrices corresponding to the k-th permutation of switches and other piecewise linear element segments.

When discretized, these equations result in:

$$\begin{aligned} x_{n+1} &= A_d x_n + B_{d1} u_n + \begin{bmatrix} B_{d2(in)} & B_{d2(no)} \end{bmatrix} \begin{bmatrix} u_{n+1(in)} \\ u_{n+1(no)} \end{bmatrix} \\ \begin{bmatrix} y_{n+1(in)} \\ y_{n+1(no)} \end{bmatrix} &= \begin{bmatrix} C_{(in)} \\ C_{(no)} \end{bmatrix} x_{n+1} + \begin{bmatrix} D_{(in,in)} & D_{(in,no)} \\ D_{(no,in)} & D_{(no,no)} \end{bmatrix} \begin{bmatrix} u_{n+1(in)} \\ u_{n+1(no)} \end{bmatrix} \end{aligned} \quad (2)$$

where:

A_d, B_{d1}, B_{d2}, C_d, D_d : discrete state matrix for the present pattern of binary switches modeled inside the group. The trapezoidal rule of integration will produce $B_{d1}=B_{d2}$. Subscript n and $n+1$ indicate the time instants.

u(in): internal sources of the state-space model. Like in standard state-space modeling, these include known forced sources and sources from non-linear current injections. (in) means internal.

u(no) : unknown sources of the state-space model at the present time n+1. This represents the nodal voltage or the current injection that can only be resolved by simultaneous solution of all groups connected to the nodes of the network. (no) means nodal.

y(in) : internal output of the model. These are usually current and voltage measurements to be taken inside the group.

y(no) : nodal output of the state-space model. This is the voltage output or current output of the group that needs to be solved simultaneously along with all groups connected to the system nodes.

The following relation can now be derived:

$$\begin{aligned} y_{n+1(no)} &= C_{(no)} \{ A_d x_n + B_{d1} u_n + B_{d2(in)} u_{n+1(in)} + \ldots \\ & B_{d2(no)} u_{n+1(no)} \} + D_{(no,in)} u_{n+1(in)} + D_{(no,no)} u_{n+1(no)} \end{aligned} \quad (3)$$

which can be viewed in two different ways:

<u>case *y(no)* is a current and *u(no)* is a voltage</u>

In this case, the equation can be viewed as a current injection known from past history and forced internal source known at present time n+1:

$$\begin{aligned} I_{hist} &= C_{(no)} \{ A_d x_n + B_{d1} u_n + \ldots \\ & B_{d2} u_{n+1(in)} \} + D_{(no,in)} u_{n+1(in)} \end{aligned} \quad (4)$$

in parallel with a discrete admittance, i.e. discrete ratio of input-output values,

$$Y = C_{(no)} B_{d2(no)} + D_{(no,no)} \quad (5)$$

This is called a *V-type SSN group* for alter discussion.

Case the output *y(no)* is a voltage and *u(no)* is a current

In this case, the equation can be viewed as a voltage source known from history

$$V_{hist} = C_{(no)}\{A_d x_n + B_{d1}u_n + \dots \\ B_{d2}u_{n+1(in)}\} + D_{(no,in)}u_{n+1(in)} \tag{6}$$

in series with a discrete impedance

$$Z = C_{(no)}B_{d2(no)} + D_{(no,no)} \tag{7}$$

In this case, the equivalent admittance is found by inversion of Z while the equivalent nodal current injection is found by the Thevenin theorem

$$I_{hist} = Z^{-1}V_{hist} \tag{8}$$

This is called a *I-type SSN group* for alter discussion.

Case the input *u(no)* and output *y(no)* are mixed type

It is also possible to derive these nodal model equations for 'hybrid' input-outputs y(no) and u(no) being composed of a mix of voltage and currents. In that case, we still have:

$$VI_{hist} = C_{(no)}\{A_d x_n + B_{d1}u_n + \dots \\ B_{d2}u_{n+1(in)}\} + D_{(no,in)}u_{n+1(in)} \tag{9}$$

and

$$YZ = C_{(no)}B_{d2(no)} + D_{(no,no)} \tag{10}$$

meaning the ratio of input to output (i.e. the discrete impedance or admittance) is no longer a pure impedance or admittance and that 'history' sources are a mix of currents and voltages.

To obtain a representation compatible with the nodal method, each input is first 'aligned' with its corresponding output, and all inputs of current type (*Ix*) are ordered before all inputs of voltage type (*Vy*).

We can then decompose the nodal relationship in the following:

$$\begin{bmatrix} Vx \\ Iy \end{bmatrix} = \begin{bmatrix} Vx_{hist} \\ Iy_{hist} \end{bmatrix} + \begin{bmatrix} z & x1 \\ x2 & y \end{bmatrix}\begin{bmatrix} Ix \\ Vy \end{bmatrix} \tag{11}$$

To insert this relationship into a nodal set of equations, one must rewrite it into:

$$\begin{bmatrix} Ix \\ Iy \end{bmatrix} = \begin{bmatrix} z^{-1} & -z^{-1}*x1 \\ x2*z^{-1} & y - x2*z^{-1}*x1 \end{bmatrix}\begin{bmatrix} Vx \\ Vy \end{bmatrix} + \\ \dots\begin{bmatrix} -z^{-1} & 0 \\ -x2*z^{-1} & 1 \end{bmatrix}\begin{bmatrix} Vx_{hist} \\ Vy_{hist} \end{bmatrix} \tag{12}$$

to produce an admittance and history current injection compatible with the nodal method. The nodal solution produces the missing voltage input *Vy* of the state-space equation (as well as *Vx*). *Ix* is found with:

$$Ix = -z^{-1}(x1*Vx - Vy) - z^{-1}*Vx_{hist} \tag{13}$$

and the group state-space equation can then be completed.

B. Simultaneous nodal voltage solution

With each nodal group having a defined 'Discrete admittance' and 'History source' for the time-step under calculation, one can now assemble these into a global admittance matrix Y_{global} and a global nodal current injection vector I_{global}.

$$Y_{global}V_{global} = I_{global} \tag{14}$$

The unknown voltage can then be determined by the solution to this algebraic equation, either by direct inversion or LU factorization, for example. This solution is then used by all groups to complete their iteration for the current time-step.

C. Complete algorithm

The complete algorithm is described in Fig 1. Basically, a time step iteration begins with all groups determining their respective operating point, corresponding set of matrices (A_d, B_{d1}, B_{d2}, C_d, D_d) and the admittance matrix *Y*. These groups compute their History Sources *Ihist or Vhist* for the current time-step. These history sources and admittance are assembled for the nodal admittance solution. The nodal voltage solution is then found, by on-line LU decomposition, from which all groups can complete their iteration by computing their internal states and outputting requested signals. Some of these outputs can be internal to the group, like current-injection non-linear models requiring a voltage as input.

The algorithm notably highlights some parallelism potential: except for the nodal solution part, all group equations can be computed in parallel ('Groupwise Decoupled Operation' in the figure). This potential is difficult to observe with regular nodal methods because of the high number of branches; in this case, the major computational burden is in the *LUV=I* solution phase, which is difficult to parallelize.

D. Application to real-time simulation

The SSN algorithm provides some flexibility with regards to size of groups and number of nodal nodes. In configurations with large groups connected by few nodal connection points, important gains can be expected when compared to state-space methods.

1) Switch management in Real-Time applications

Consider, for example, a very large lumped network with *x* state variables, depicted in the upper part of Fig. 1. In a classic state-space approach, we would obtain one large set of ABCD state-space matrices with x states. Considering a state-space solver method that pre-computes the matrix sets with regards to switch position permutations, excellent speed-performance can be expected. However, pre-calculation of the state-space matrix has practical limits: the number of matrix sets that can be pre-computed and stored is limited by the available memory. In this example, a set of 2^6=64 different state-space matrices must be pre-computed.

With a mixed method composed of nodal and state-space solutions, important optimizations can be obtained. Consider the same system with a three-phase connection point in the middle (Fig. 1, lower part) using the SSN method.

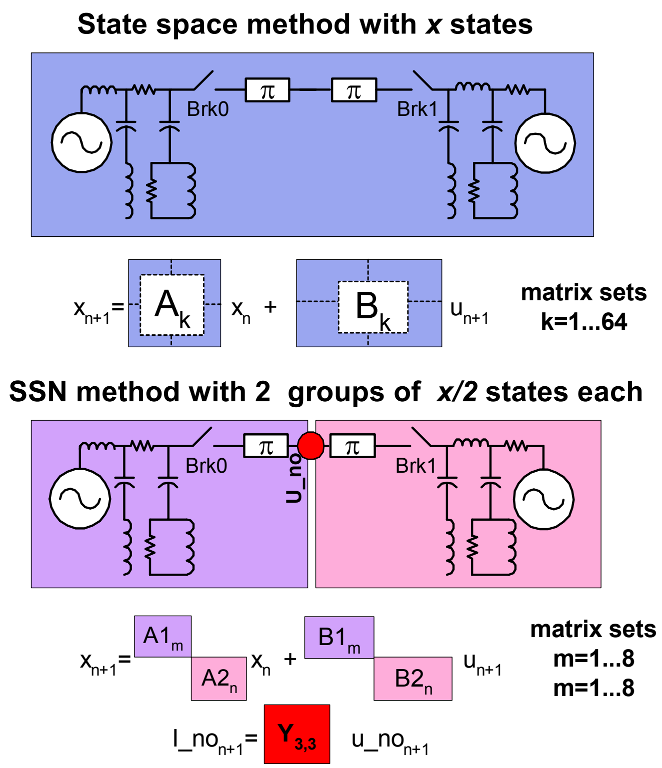

Fig. 1 Large network state-variable allocation: states-space (upper) , state-space nodal (lower)

In the SSN method, two SSN groups separated by 3 nodal connection points (separating the complete network in two equal part let say) can be defined. In this case, the algorithm would imply 2 state-space system iterations with *x/2* state variables each, plus a 3x3 nodal matrix inversion (for the sake of clarity, the dependence of the 2 pi-line section capacitance at the nodal junction is neglected). This would cut approximately by half the total number of operations required by the state-space iterations and add a 3x3 matrix inversion. Indeed, Fig. 1 shows that the SSN method makes the complete state-space equations block-diagonal by virtue of the decoupling induced by the approach. The decoupling is partial, however, as the nodal solution links all parts of the network equations. Using a single CPU approach, the computational gain can approach a factor of 2 if the state-space iteration becomes more important, in comparison to the same 3x3 matrix inversion.

Furthermore, the problem of memory storage of switch permutations is solved here: each group contains only the pre-calculated set of matrices for the switch contained within the groups. Taking again the specific example of Fig. 1, which is composed of two three-phase breakers, full pre-calculation of circuit modes in the standard state-space approach would require the storage of 2^6=64 permutations of states-space equations of *x* states. In the SSN approach, two sets of 2^3=8 system matrices need to be stored (one for each group), thus drastically reducing memory requirements.

Separation of switches is always possible because they can be modeled as a separate group in the proposed method. In that case, only the D matrix subsystem is non-empty, and the group admittance is included directly in the global admittance matrix in way similar to standard nodal method.

2) *Facilitating parallel computations*

As already mentioned, the 2 example groups are partially independent at the algorithmic level. They only depend on each other at the nodal solution stage. This allows a parallel calculation of the example group equations on two CPUs or cores. As the group size becomes larger, making the 3x3 nodal solution burden negligible compared to the state-space iterations, and if inter-CPU communication time is neglected, a speed-up gain of 4 can be achieved in this example, compared to the standard state-space approach.

The system could also be 'grouped' in the same way as the classic nodal method, which will result in a large number of group (i.e. branches) updates and a nodal solution for an admittance matrix of large size N with *many* groups of few states each. Optimization between the number of nodes and the size of state-space groups can be studied in the general case.

E. Usage of Higher-Order L-Stable Discretisation Technique to Increase Numerical Stability and Accuracy

It is well-known that the exact solution to the state-space equation (Eq.1) is equal to:

$$x_{n+1} = e^{Ah}x_n + \int_t^{t+h} e^{A(t-\tau)}Bu(\tau)d\tau \qquad (15)$$

where *h* is the discretisation time step. It should be recognized that 2 distinct approximations are necessary to obtain a numerically computable expression:

1- The approximation to the matrix exponential e^{Ah}
2- The way the input *u* is approximated during integration

The traditional EMTP approach uses the trapezoidal approximation (Padé 1,1) of the matrix exponential, equal to:

$$e^{Ah} \cong \frac{I + hA/2}{I - hA/2} \qquad (16)$$

combined with a linear interpolation of the input during the integration step. The trapezoidal rule is however unstable during fast disturbances, therefore a special method called CDA is applied when a disturbance is detected. During CDA steps of EMTP, the Backward Euler method is used for both matrix exponential and input terms, in addition to a time-step change in the original implementation [11].

Using a higher order in Equation 15 can lead to interesting results especially with regards to stability issues. For example, the ARTEMiS 'Art5' solver, based on the (2,3)-Padé order 5 approximation of the matrix exponential, of formula equal to

$$e^{Ah} \cong \frac{I + 2hA/5 + (hA)^2/20}{I - 3hA/5 + 3(hA)^2/20 - (hA)^3/60} \qquad (17)$$

has a property called L-stability, an extension of A-Stability, which makes it immune to the kind of numerical instability of the trapezoidal rule[3][9][10]. It should be noted that the Backward Euler rule is also a L-stable Padé approximations.

In real-time applications, CDA is avoided in its original implementation because of the time-step change. Constant time-step CDA can be performed to remedy this in real-time applications, but is rarely used in practice. Nevertheless, a potential problem of CDA remains in cases where parallelizing the SSN algorithm for its groupwise operation is desired. In such cases, CDA detection is a global operation that affects all groups. Replacement of CDA with L-stable discretisation approaches can help to keep the original granularity of the SSN approach.

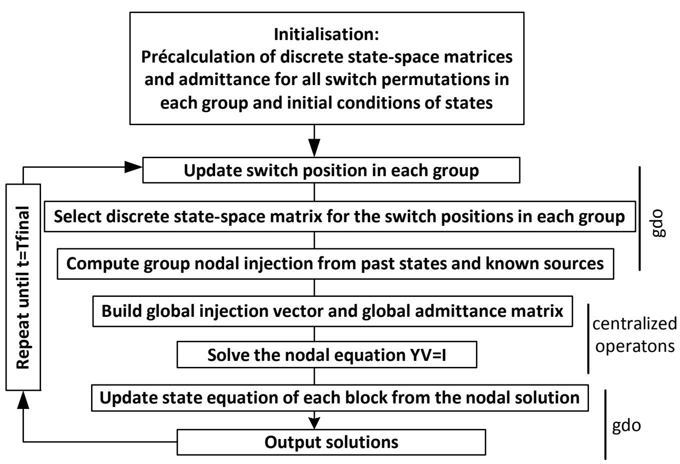

Fig. 2 SSN algorithm (real-time loop)

III. Validation

The SSN method has been validated on several models of varying complexity. The method has been implemented for parallel real-time simulation with RT-LAB and SimPowerSystems for Simulink using a 'C'-code S-function that substitutes itself for the regular SPS solver S-function.

A. *Simple time-segment linear system*

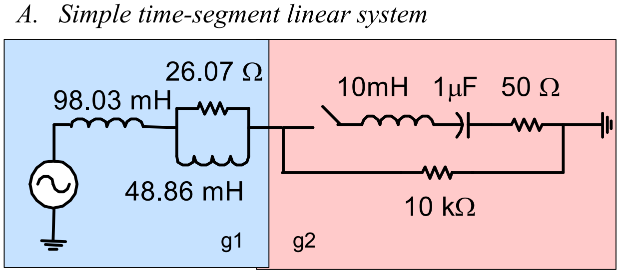

Fig. 3 Switched RLC circuit

A simple switched RLC circuit is used to test the SSN algorithm with regards to numerical oscillations. The circuit is separated into 2 SSN V-type groups (blue and red in Fig. 3), because they are of inductive type. During steady-state conditions, the switch is closed at 0.05 seconds. The result of the SSN simulation is compared with the native SPS Tustin solver and ARTEMiS L-stable order 5 discretization. Also, the following solvers and simulation program where used.

- SSN (trap): trapezoidal rule of integration for both exponential and input terms
- SSN (5g,1n): Padé (3,2) approx of matrix exponential with Backward Euler method for the input term.
- SSN(5g,2n) : Padé (3,2) approx of matrix exponential with Trapezoidal method for the input term.
- SSN (trap+CDA): SSN (trap) with 1 Backward Euler step at discontinuities.
- SPS: (State-space approach) Tustin rule which is equivalent to the trapezoidal rule of integration.
- ARTEMiS: (State-Space approach): Padé (3,2) approximation with linear interpolation of input term.
- EMTP-RV: Trapezoidal rule of integration. The one-step CDA option was used with this solver.

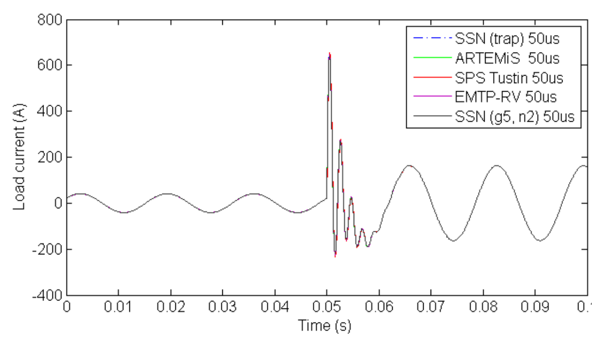

Fig. 4 RLC circuit load current

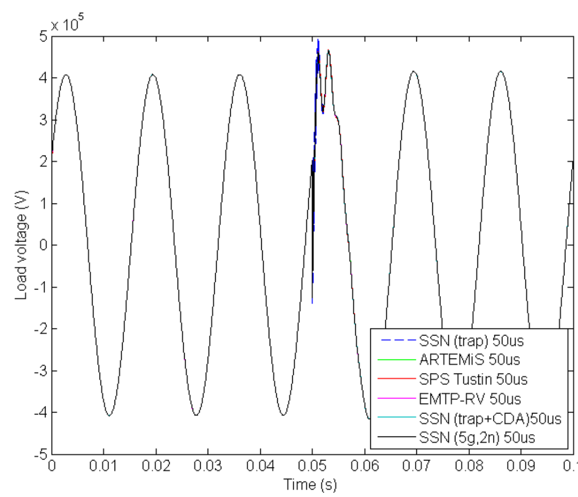

Fig. 5 RLC circuit load voltage

The results for the current response of Fig. 4 and Fig. 5 match well across all solvers. More interesting is the voltage zoom at the switching. In this case, some voltage oscillations are present in the SSN(trap) response in Fig. 6, but are not present for either SPS, ARTEMiS, EMTP-RV or SSN(trap+CDA) responses. This is a well-known numerical effect of the nodal method for which solutions have been published [11]. If CDA is applied for one-step in the SSN approach (SSN trap+CDA), the oscillation disappears. The use of the SSN (5g, 2n) solver is interesting in this aspect because it dampens the oscillation without any solver change, an important aspect when choosing a real-time simulation method.

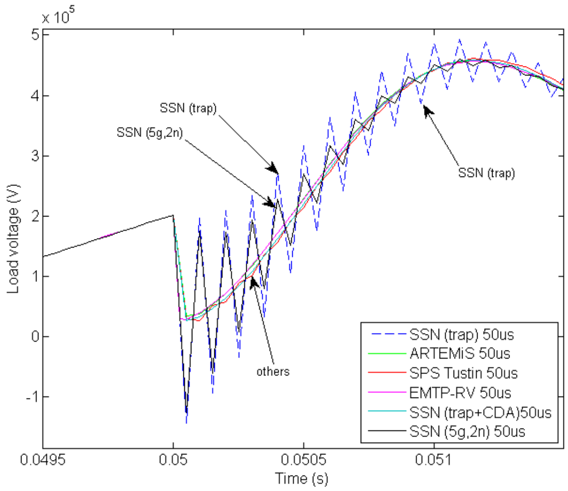

Fig. 6 RLC circuit load voltage (Switching instant zoom)

If the 10kW of the model is completely removed, undamped numerical oscillations occur for the voltage with the SSN (trap) approach (the current response is not changed). This oscillation adds-up at each switching action, as illustrated in the example of Ref [11]. The SSN(5g,2n) damps this oscillation and prevents it from build-up. It is interesting to note that the circuit is very close to having a state-variable dependency between the source inductance and the RLC group inductance. The dependence is total if the 10kW resistor is removed.

In pure state-space approaches, this dependency is resolved before determination of the ABCD system matrices. In a nodal approach, there is no a priori solution to this because the branch equations are discretized separately.

B. Complete 12 pulse HVDC system with Inlined Thyristor Valve compensation

The SSN method implements 'Inlined Thyristor Valve Compensation' algorithm (ITVC). The ITVC method is used to maintain the simulation accuracy even when the thyristor switching occurs between time-steps. The term 'Inlined' comes from the fact that ITVC is built into the SSN algorithm with a single line of code which means that the computational burden is negligible. It must be understood that simulation imprecision, in a case where the ITVC algorithm is turned off, is caused by fixed step sampling and would be similar in any fixed step software like EMTP, PLECS or PSCAD.

This test models a 1000 MW (500 kV, 2 kA), HVDC link used to transmit power from a 500 kV, 5000 MVA, 60 Hz network to a 345 kV, 10 000 MVA, 50 Hz network.

The rectifier and the inverter are 12-pulse converters. The rectifier and the inverter are interconnected through a 300 km distributed parameter line and two 0.5 H smoothing reactors. The transformer tap changers are not simulated and fixed taps are assumed. The tap factor used on the primary voltage is 0.90 on rectifier side and 0.96 on inverter side. Reactive power required by the converters is provided by a set of capacitor banks plus 11th, 13th and high pass filters for a total of 600 MVAR on each side.

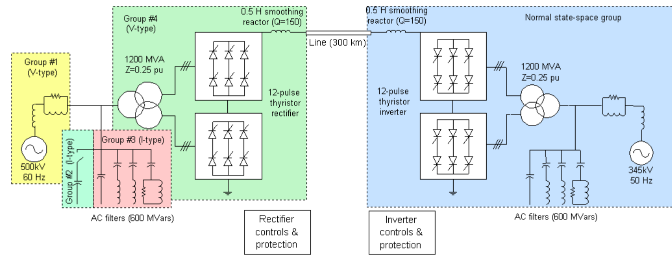

Fig. 7 12-pulse HVDC system

On the rectifier side, the capacitor of the filter bank is split into two parts, one of which is switched. For the purpose of the test, groups were created on the rectifier side in the following manner:

Group 1: AC-source and impedance (V-type SSN group)

Group 2: Switched capacitor (I-type SSN group)

Group 3: Fixed filter bank (I-type SSN group)

Group 4: Transformer and thyristor rectifier and smoothing rector (V-type SSN group)

The inverter side is simulated using standard state-space modeling of ARTEMiS (order 5 discretisation) [3] when using the SSN method (right side of Fig. 7, in blue)

The test consists of the energisation of the DC link to the nominal current.

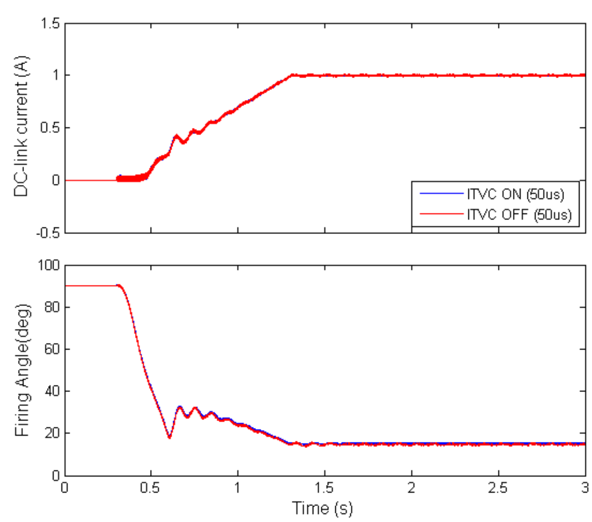

Fig. 8 DC-link current during HVDC link energization (see the zoom on next figure)

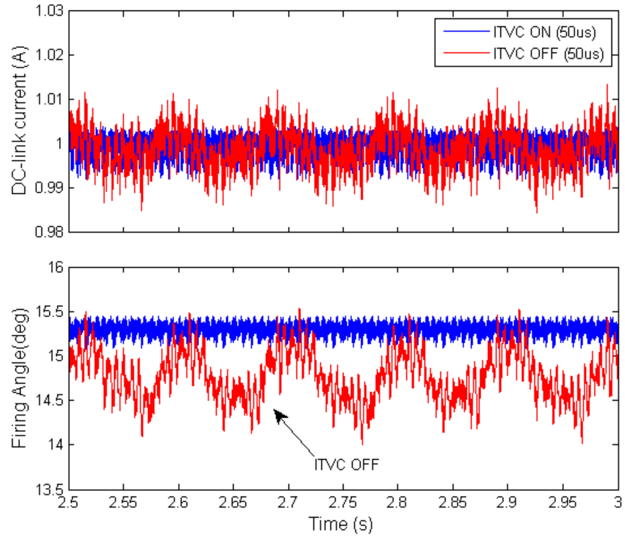

Fig. 9 DC-link current (zoom)

Examination of Fig. 9 clearly shows the effect of the ITVC method as the current jitter completely disappears using the method.

C. HVDC system with several switched AC filter banks

The same HVDC model can be simulated with additional switched capacitor banks on the AC bus. In the SSN method this comes at a very low cost because all the banks are connected to the same nodes. In SSN, the rectifier side is separated into SSN groups as shown in Fig. 10. Notably, each switched filter bank makes up a group and all these groups are connected to the same nodes, thus the admittance matrix remains small. This results in a system with a nodal matrix of rank 11, connecting a total of 12 groups.

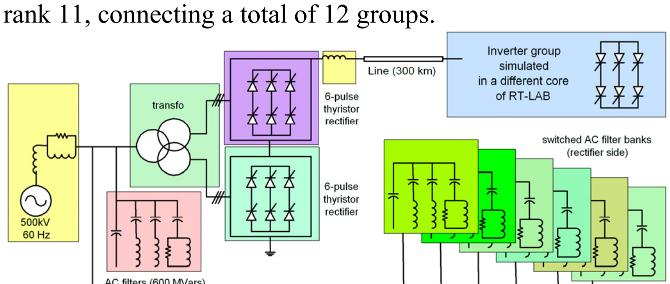

Fig. 10 HVDC with switched AC capacitor banks (24 thyristors + 18 breakers in total)

The test consists of the energisation of the DC link to the nominal current with a 300MVA capacitor switched ON at 1.5 seconds in the simulation, then a 2nd bank is turned ON 0.3 seconds later, and so on until 5 banks are switched ON.

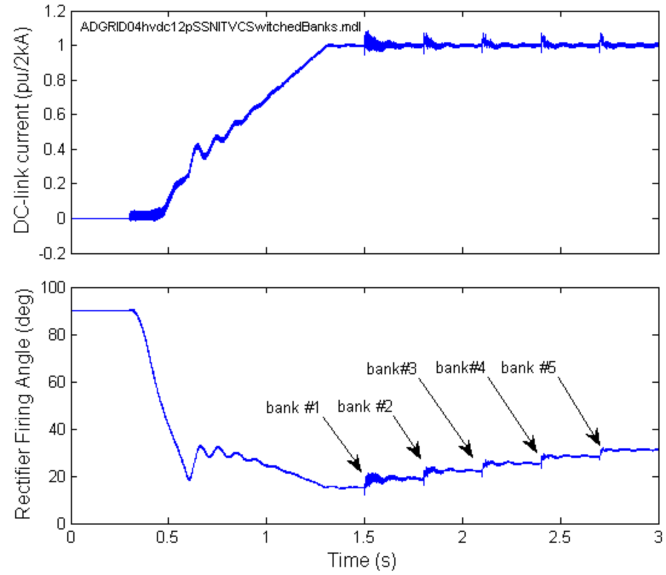

Fig. 11 DC-link current and firing angle during HVDC link energization and AC filter bank commutation.

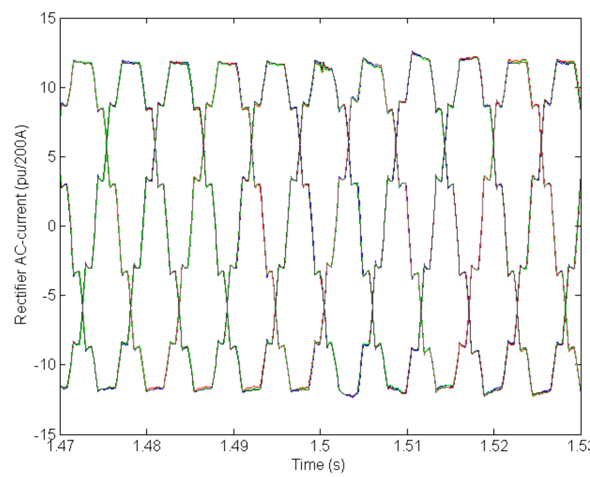

Fig. 12 Rectifier AC-current going into the transformer, zoomed near the capacitor switching (all solvers).

The simulation results, shown in Fig. 11 and Fig. 12 show the transients caused by the filter bank switching, and also the rectifier firing angle which increases each time a bank is connected. Simply explained, the filter banks inject reactive power at the AC-side, causing an increase in AC voltage which the rectifier regulator responds to by increasing the firing angle. This maintains the DC-link current at the prescribed level.

D. Static VAR Compensator (SVC) real-time simulation

A Static VAR Compensator is simulated using the SSN method. The SVC is composed of one Thyristor Controlled Reactor (TCR) and 3 Thyristor Switched Capacitor (TSC) banks, for a total of 24 thyristors. The selection of SSN groups comes naturally in this case: the TCR and feeding network form one SSN group and the TSC form 3 other groups. What is particular about this group selection is that it produces a nodal matrix of rank 3 since all groups are connected to the same nodes.

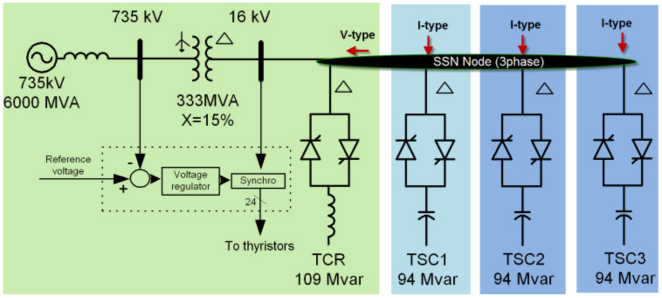

Fig. 13 SVC model (24 thyristors in total)

An interesting aspect of the real-time simulation of this type of circuit is the linearilty of control of the TCR. As explained previously, the fixed time-step frame causes a rounding effect in the firing time of the thyristors. The ITVC method is therefore used to compensate for this effect.

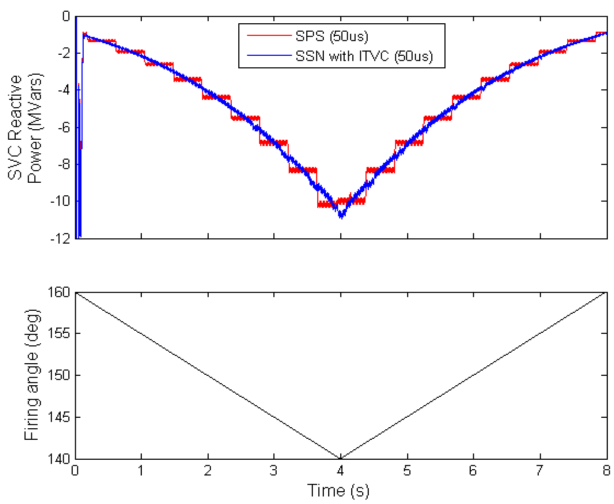

Fig. 14 SVC firing angle scan test results

To better explain this problem, a test was performed in which the firing angle was slowly scanned from 160° to 140° and back, and the SVC reactive power output was plotted in Fig. 14. In the figure, a quantization is observed of the reactive power output when ITVC is not used, as in standard SPS solver (or any other fixed-step solver such as PLECS, EMTP, PSCAD). With ITVC, the reactive power output is locally linear. A particularity of this model is that the groups must be discretized with the Art5 solver to obtain a stable scan.

E. Small network breaker test model

This test case is a small network used to test breaker action when various fault contingencies occur. It is a 3-phase system composed of two sources connected through 4 short pi-lines (9-11km). The lines are protected by breakers at both ends and various switched faults (Def_X) are included in the model to apply faults in various sequences. The one-line model diagram is depicted in Fig. 15. This model is difficult to simulate in real-time because it contains several switches (22 in total) and the line length is too short to be used to decouple the computation of subsystems at both ends; a technique commonly used by most real-time simulators. In the state-space approach of SPS and ARTEMiS, precalculation and storage of state-space matrices for all switch patterns is preferred in real-time applications because on-line state-space matrix calculation is relatively slow. When many coupled switches are present in the complete system, the number of combinations can be too great for available memory to support. This type of problem does not occur when using nodal approaches if on-line decomposition by LU method of the admittance matrix is done[16].

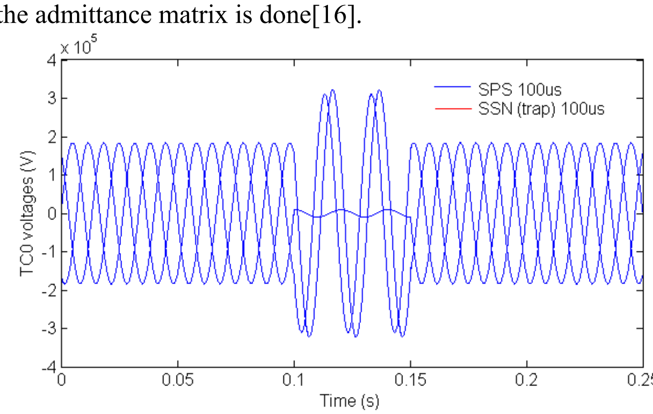

Fig. 16 Phase voltages at the input of TC0

In the SSN approach, this problem is solved because the switches can be located in different groups. Groups precompute their state-space systems for all switch combinations independently of each other. Each group then provides the pre-compute local admittance to the global nodal solver which finds the simultaneous solution for all nodes.

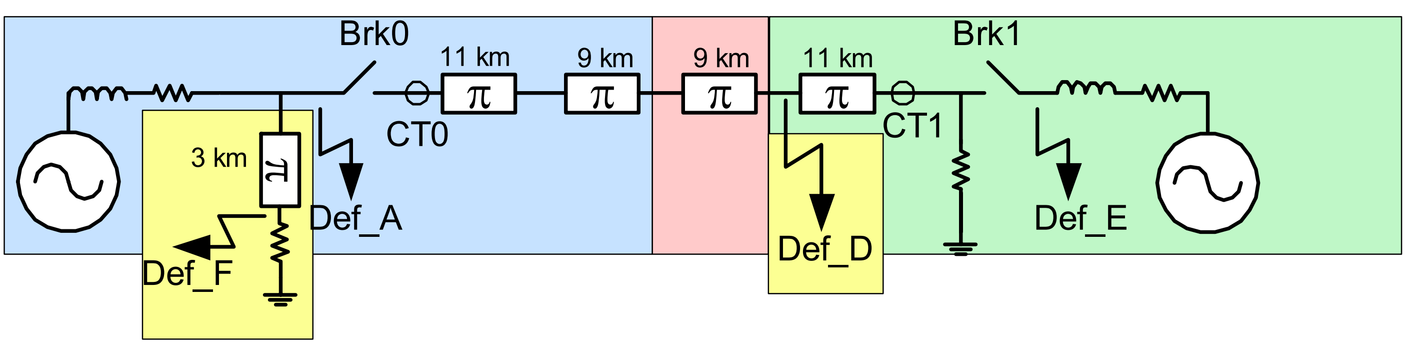

Fig. 15 Breaker test model (22 switches in total)

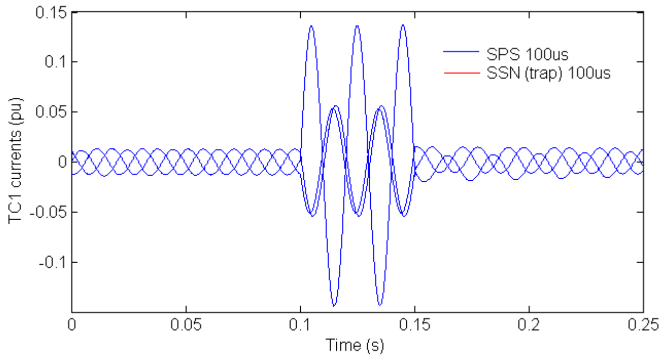

Fig. 17 TC1 current during fault.

On-line decomposition by the LU method is also used in SSN for the nodal part.

For the purpose of our test, the distribution system was decomposed into 5 SSN groups (as highlighted in Fig. 15) linked by 9 nodal points and simulated at a time step of 100 µs. Each group contains a maximum of 6 switches, so precalculation of the state-space matrix is not problematic. Also, the group containing 2 pi-lines is modeled using the 'mixed-type' SSN interface of Section II. A single-phase fault at point 'Def_D' is applied with closed breakers for the purpose of the test. Fig. 16 shows the voltage at entry of the current transformer TC0 for both the SSN method and native SPS method. The results match perfectly.

F. Frequency-Dependant (Marti-type) line model

The SSN method has an advantage in that it can be coupled with complex models that are more efficiently computed in a nodal approach, like the Frequency Dependant Parameter Line model (FD- line model) [14][15].

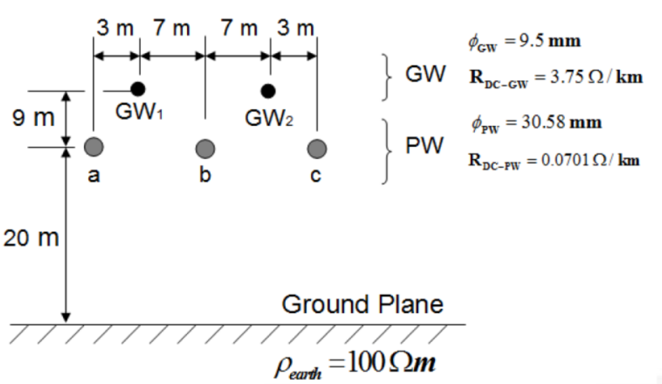

Fig. 18 FD-line geometric configuration

A simple test was performed on a 191 km 3-phase FD-line with the geometric configuration of Fig. 19. The line model was fitted with a total of 39 poles of impedance function fittings and total of 67 poles for the propagation functions. The line was energized with two receiving phases opened and the third with a 1Ω resistance. Simulation results perfectly match those generated using EMTP-RV.

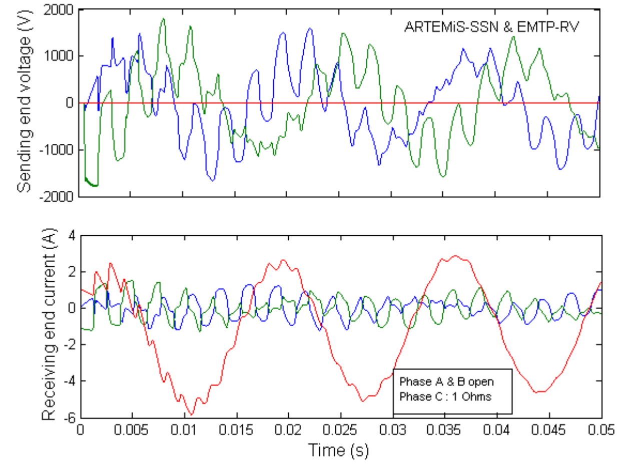

Fig. 19 FD-line model simulation

The SSN algorithm now supports the FD-line model included with the SimPowerSystems blockset. Wide-Band (or Universal line model) will be supported later in 2010.

IV. Real-Time Simulation Results

The various models presented in this paper have been tested in real-time on a RT-LAB real-time simulation target platform [7], comprised of a single 3.2 GHz Xeon i7 Quad-core PC running under a RedHat Linux kernel. Also, these tests use the SPS implementation of the SSN algorithm.

The HVDC system was simulated with 3 cores: one core for the rectifier side based on the SSN method, the second core for the inverter side based on the state-space approach and the third core was used for simulating the HVDC controls. The worst case time-step achieved was 10 µs with the groups identified in Fig. 7.

HVDC-*6*fb is the result of real-time simulation with 6 3-phase switched filter banks. This shows the capacity of the SSN algorithm to run in hard real-time with minimal overhead with regards to the total number of switches. Separate grouping of the two 6-valve groups could have been made to reduce the number of pre-calculated matrix sets, as the proposed SSN algorithm offers this flexibility. The HVDC-6fb timing was done with ITVC compensation turned ON, as this comes at negligible computational cost.

The Breaker test setup was simulated on a single core. The worst case condition produces a time-step of 21 µs.

TABLE I Hard real-time time-step results in RT-LAB

Test case	SSN time-step (µs)	CPUs used (power+ctrl)
HVDC	10	2+1
Breaker test	21	1
HVDC-6fb	37	2+1
SVC	25	1+1
FD-line	6	2

The above table summarizes the real-time results. It gives the maximum calculation time of all time-steps, a condition suitable for 'hard' real-time simulation. The switching events maximize processor un-caching effects. The measurements shown in the table below were performed without I/O devices.

V. CONCLUSION

This paper has presented a power system simulation solver that uses a unified approach based on state-space equations and nodal admittance approach modeling. The approach makes use of internal grouping of electric elements to enable a modulation of computational burden between state-space and nodal approaches. HVDC with switched filter, SVC and breaker test systems have served to validate the approach.

The method is currently implemented inside the SimPowerSystems blockset as a 'C' S-function to run the algorithm in real-time in the RT-LAB platform[7].

An interesting aspect of the method is that it favors neither the Norton nor Thevenin approach. Instead, it uses both, even in combination within a group. Further studies are required to characterize this aspect of the algorithm with regards to accuracy for example.

The choice of groups and nodal voltage points is another aspect that requires further study because optimal choices may be found between the size of groups vs. size of the nodal matrix, leading to faster simulation, a key aspect in many real-time applications.

The inclusion of the SSN algorithm in SimPowerSystems has the advantage of making all the MOV, saturable transformer and machine models available within the approach because these models are automatically included in the group equations by the SPS routines. Implicitly discretized machine models typically provide better numerical stability and are currently under development in the SSN algorithm [17].

Finally, the proposed SSN approach also enables coupling into a state-space method of complex models, like the FD-line, that are naturally more efficient when coded in the nodal approach.

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